B.Sc. Degree I (Hons.)

Paper I Matrices

Orthogonal matrix

Let A be a square matrix.
Then A is said to be orthogonal if $AA' = I$ (unit matrix) $= A'A$

Theorem Every orthogonal matrix is invertible.

Soln Let A be an orthogonal matrix.

$\Rightarrow AA' = I$. ——— (1)

We've to prove that
A is invertible, i.e. A^{-1} can be obtained, i.e. $|A| \neq 0$

From (1) $AA' = I$

$\Rightarrow |AA'| = |I|$

$\Rightarrow |A||A'| = 1 \quad (\because |I| = 1)$

$\Rightarrow |A||A| = 1$

$\Rightarrow |A|^2 = 1 \Rightarrow |A| \neq 0$

So, A is invertible.

Example

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\Rightarrow A' = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\therefore AA' = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \sin\theta\cos\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta - \sin\theta\cos\theta & \sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$\therefore$ A is an orthogonal matrix